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well to equation (2). For example, (2) is not true if arcsin 3/5 and arcsin 8/17 are angles whose terminal lines lie in the second quadrant.

Direction is given in some instances to prove the statement (1) by taking the tangent of both sides. Without some restrictions on the angles involved this does not prove the equality; else

$$60° = 240°$$

because it happens that

$$\tan 60° = \tan 240°.$$

Is it any longer a mystery why the student does not obtain a better grasp of his mathematics?

III. The Path of a Projectile when the Resistance Varies as the Velocity.

By E. L. Rees, University of Kentucky.

The very simple problem of finding the path (and its hodograph) of a projectile in a vacuum is treated in most elementary treatises on vector analysis, but the problem of finding the trajectory when the body moves in a resisting medium is usually not considered as the general problem does not lend itself readily to vector treatment. However, there is a special case of some interest which presents no difficulties. If we assume that the resistance varies as the velocity (which under certain conditions is approximately true for low velocities in the air) the vector differential equation of motion is of a very simple type and its solution differs in no essential way from that of a scalar differential equation of the same type. The equation of such a motion is

$$\frac{d^2\boldsymbol{r}}{dt^2} = \boldsymbol{g} - k\frac{d\boldsymbol{r}}{dt},$$

where $\boldsymbol{g}$ is the vector acceleration due to gravity, and k is a positive scalar const. This is a linear vector differential equation which may be integrated at once by applying the integrating factor e^{kt}.

Integrating we get

$$\frac{d\boldsymbol{r}}{dt} = \frac{\boldsymbol{g}}{k} + e^{-kt}\boldsymbol{C}_1$$

and

$$\boldsymbol{r} = \frac{\boldsymbol{g}t}{k} - \frac{e^{-kt}}{k}\boldsymbol{C}_1 + \boldsymbol{C}_2,$$

where $\boldsymbol{C}_1$ and $\boldsymbol{C}_2$ are the vector constants of integration.

Assuming $\boldsymbol{r} = 0$, $\boldsymbol{v} = \boldsymbol{v}_0$ when $t = 0$ we have

$$\frac{d\boldsymbol{r}}{dt} = \frac{\boldsymbol{g}}{k} + e^{-kt}(\boldsymbol{v}_0 - \boldsymbol{g}/k) \tag{1}$$

and

$$\boldsymbol{r} = \frac{\boldsymbol{g}t}{k} + \frac{1}{k}(1 - e^{-kt})(\boldsymbol{v}_0 - \boldsymbol{g}/k). \tag{2}$$

The first of these equations is the equation of the hodograph and the second is the vector equation of the trajectory.

The highest point reached by the projectile may be found by multiplying the first equation dotwise by $\boldsymbol{g}$ and setting $\boldsymbol{g}\cdot(d\boldsymbol{r}/dt) = 0$. Solving this equation for t we find $t = 1/k \log(1 - k\boldsymbol{v}_0\cdot\boldsymbol{g}^{-1})$, and this substituted in equations (1) and (2) gives the velocity and the position vector of the projectile at the highest point. Similarly, to find the time of flight and the range on the horizontal plane through the point of projection multiply the second equation by $\boldsymbol{g}\cdot$ and set $\boldsymbol{g}\cdot\boldsymbol{r} = 0$, solve for t, etc.

Since k is positive and t varies from zero to infinity e^{-kt} varies from 1 to 0 and the hodograph is seen to be the segment of a line which terminates at the tips of the vectors $\boldsymbol{v}_0$ and $\boldsymbol{g}/k$.

If the initial velocity is allowed to take all possible directions the hodographs will form a bundle of line segments extending from the tip of $\boldsymbol{g}/k$. If the initial speed is constant the segments will terminate in the surface of a sphere with center at the origin and radius v_0. If the angle of projection varies in a vertical plane and the initial speed is constant the segments will form a pencil one end of each segment resting on the tip of $\boldsymbol{g}/k$ (vertex of pencil) and the other on a circle of radius v_0. The vertex of the bundle or pencil corresponds to the limiting velocity which is the same for all initial velocities.

The cartesian equation of the trajectory is found from the vector equation in the usual way to be

$$y = \frac{g}{k^2}\log(1 - x/l) + \frac{mx}{l},$$

where

$$l = \frac{v_0 \cos\alpha}{k} \quad \text{and} \quad m = \frac{v_0 \sin\alpha}{k} + \frac{g}{k^2},$$

α being the angle of projection.

From this equation it is seen that the trajectory has an asymptote whose equation is $x = l$, which accords with the known fact that the trajectory of a body moving in any resisting medium has a vertical asymptote.

RECENT PUBLICATIONS.

REVIEWS.

CAJORI'S HISTORY OF MATHEMATICS.

A History of Mathematics. By FLORIAN CAJORI. New York, The Macmillan Company, 1919; pp. x + 514. Price $4.

It is twenty-five years since Professor Cajori, then already known for his contributions in the field of mathematical history, published the first edition of the work under review. During this period the book has been reprinted six times, and the author and publisher have now united in giving to students a second edition, to a considerable extent rewritten and in certain chapters materially enlarged.